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**Developing Analysis, Modeling, and Simulation (AMS) Tools for Connected and Automated Vehicle (CAV) Applications**

**Algorithm Description Document: Coordinated Merge Model**

SI Conversion Chart. See https://www.fhwa.dot.gov/publications/convtabl.cfm for html version.

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List of Abbreviations

ACC adaptive cruise control

ADAS advanced driver assistance systems

AMS analysis, modeling, and simulation

AV automated vehicle

CAV connected and automated vehicle

CACC cooperative adaptive cruise control

CV connected vehicle

CM coordinated merge

IDM intelligent driver model

NGSIM Next Generation SIMulation

TMC traffic management center

TTI travel time index

UAV unmanned aerial vehicle

USDOT United States department of transportation

V2V vehicle-to-vehicle

V2I vehicle-to-infrastructure

Executive Summary

It is known that merging maneuvers cause traffic disruption and result in shockwave formation and propagation. This study proposes a coordinated merge (CM) model which jointly optimizes the lateral trajectory of a lane-changing vehicle and the longitudinal control of an impacted vehicle in the target lane. The objective is to minimize the disturbance caused by the merging process in the target lane. Therefore, we show how to predict the impact of the lane-changing on a platoon of vehicles. Towards this objective: i) we first design a model predictive controller (MPC) for the vehicle in the target lane to properly respond to a discretionary lane-changing maneuver; ii) we then specify a set of possible lateral trajectories that may be adopted by the lane-changing vehicle; iii) we then compute the required deceleration of the vehicle directly impacted by the lane-changing based on the estimated set of the lateral trajectories; iv) we finally choose the trajectory that minimizes the total deceleration effort(i.e., minimizes the disturbance in the platoon of vehicles). Analytical and simulation-based investigations are performed to assess the capability of the proposed approach in minimizing the acceleration disturbance. We assume the availability of Vehicle-to-Vehicle (V2V) communication. Therefore, the lane-changing vehicle and the vehicle directly impacted by this maneuver in the target lane can share their acceleration profile. The simulation results show that communicating the information on the lane-changing maneuver can help connected and automated vehicles in the target lane execute more reliable and efficient maneuvers.

Another objective of this document is to help future users easily to adapt and customize this CM model in a traffic simulation tool they preferred to meet their simulation needs. To this end, this document describes the algorithms/logic of this model in detail. It also illustrates how this model was developed, calibrated, and validated. The pseudocode of this model was included in the appendix

Chapter 1. PURPOSE OF THIS MODEL

Purpose of this Document

Connected autonomous vehicle (CAV) technologies offer potentially transformative societal impacts, including significant mobility, safety, and environmental benefits. The United States Department of Transportation (USDOT) has led the development, research, and standards-making to support these technologies and is currently developing deployment and implementation approaches and guidelines.

For CAV applications to be deployed, state and local transportation agencies must first be able to effectively and fully quantify the impacts of such deployments and identify which application best addresses their unique transportation problem. Traffic analysis, modeling, and simulation (AMS) tools provide an efficient means to evaluate transportation improvement projects before deployment. Current AMS tools are not well-suited for evaluating CAV applications due to their inability to incorporate vehicle connectivity/communication and automated driving features. To mitigate this gap, the Federal Highway Administration (FHWA) has sponsored this project to develop CAV applications/models based on field data to support the CAV simulation community. Three CAV applications were developed under this project. They are a lane changing (LC) model for light-duty CAVs, a combined application model that integrates speed harmonization (SH) and coordinated merge (CM), and an improved cooperative adaptive cruise control (CACC) model for light-duty CAVs.

This document presents the CM model of the joint application in detail. The objective of this document is to provide detailed informationabout this model to improve the CAV simulation community. This document is expected to help future users easily to adapt and customize this model in a traffic simulation tool they preferred to meet their simulation needs. To this end, this document describes the algorithms/logic of this model in detail. It also illustrates how this model was developed, calibrated, and validated. The pseudocode of this model was included in the appendix.

Purpose of this Model

To illustrate the importance of considering both longitudinal and the lateral movements, this study focuses on lane-changing maneuvers at merge locations; lane-changing maneuvers can significantly impact traffic flow by creating disturbances in both initial and target lanes potentially resulting in shockwave formation and propagation. Depending on the lane-changing trajectory and the traffic conditions in the origin and target lanes, the significance of the disturbance (i.e., shockwave magnitude and duration) can change. For instance, a sudden lane-changing maneuver in the synchronized flow regime can create a more severe shockwave compared with the same lane-changing maneuver effect in the free-flow regime. However, this study intends to show that with careful coordination between lane-changing and longitudinal movements at the merge location, one can potentially eliminate the shockwave formation or reduce its magnitude to a great extent.

In line with the above goal, this section proposes a combined optimization approach for longitudinal and lateral vehicular movements to minimize the disturbance caused by a lane-changing maneuver at a merge location. The approach relies on (1) designing a Model Predictive Controller (MPC) for the car-following behavior of the vehicles directly impacted by the lane-changing maneuver to create a reliable response to this maneuver and (2) optimizing the lane-changing trajectory based on the following considerations:

* The trajectory should minimize the changes in the average velocity in the target lane (i.e., the magnitude of a traffic disturbance);
* The entire lane-changing maneuver should have a reasonable duration; and
* The new follower behaves based on the MPC.

Accordingly, the key contributions of this study are to introduce an MPC along with a class of trajectories for automated vehicles' discretionary lane-changing maneuvers that minimizes the traffic flow disturbance in the target lane, while taking into account the vehicle dynamics and the passengers' comfort limitations. The MPC works by the lane-changing trajectory to minimize the disturbance associated with the lane-changing maneuver. Therefore, this study offers a joint optimization of the discretionary lane-changing and car-following maneuvers.

Document Overview

This document will introduce a new merge coordination model capable of significantly reducing or eliminating shockwaves at merge locations. In the following sections, we introduce the model details and the logic behind each element in the model. Model calibration and validation is discussed next. This section is followed by a simulation-based analysis of the impacts of the proposed model on traffic flow dynamics. The document is concluded with a summary of the findings.

Chapter 2. MODEL DEVELOPMENT AND LOGIC

This section discusses the model development procedure and provides details about the overall design of the merge coordination model.

Descriptions of Model Logic

The merge coordination model consists of two core modules: car-following and lane-changing. In the following, the logic behind both modules is presented.

Car-following behavior

Interests in vehicle automation started more than a few decades ago. From the car-following behavior perspective, vehicle automation studies have mainly focused on vehicle platooning and its various derivations. Vehicle platooning, and platoon control design are considered necessary to achieve connected automated transportation system. Moreover, the role of communications (vehicle-to-vehicle and vehicle-to-infrastructure communications) on the performance of the platooning/car-following behavior has been investigated extensively.

Accordingly, several platooning strategies have been proposed in the literature. These strategies can be categorized into three distinct categories: (1) constant spacing, where the platoon controller is focused on keeping a fixed spacing between vehicles; (2) constant time-headway, where the platoon controller is focused on keeping a fixed time-headway between vehicles; and (3) variable time-headway, where fluctuations in time-headway are allowed to dampen the disturbances in the platoon. In general, the effectiveness of a platooning strategy and its associated controller have been investigated based on the concept of string stability. String stability is concerned with how the disturbance propagates through the string (Swaroop, 1997). In a string stable platoon, any disturbance has to attenuate along with the platoon and should not stay for a long time. Several studies investigated string stability in Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) systems. In Seiler et al. (Seiler et al., 2004), by thinking of a vehicle as a system, the author describes how the disturbance caused by a vehicle propagates in the platoon. This type of research framework is mostly based on dealing with transfer functions between initial disturbance and other states affected by the disturbance such as position error or acceleration of followers. Many studies have adopted a similar approach to investigate string stability. For example, Maschuw et al. (Maschuw et al., 2008) and Kianfar et al. (Kianfar et al., 2014) investigated how a disturbance affects the acceleration of followers. Moreover, these studies proposed a methodology to reduce the overshoot in followers' velocity and acceleration caused by the disturbance based on synthesis framework.

In addition to the platooning strategies that only rely on onboard sensors, several studies have explored the impact of vehicle-to-vehicle (V2V) and vehicle-to-Infrastructure (V2I) communications and the information from other vehicles and infrastructure on the string stability. In Swaroop (Swaroop, 1997), information about the platoon leader and the predecessor were found critical for string stability in CACC. Moreover, Swaroop showed that communications with the platoon leader are the necessary condition to ensure string stability in a platoon controlled by the constant spacing policy. Seiler et al. (Seiler et al., 2004) showed that the availability of the leader information is critical to ensure string stability. They also compared the platooning results with the case without leader information. Moreover, Wang et al. (Wang and Rajamani, 2004) compared constant time headway and constant spacing strategies. Recently, Talebpour and Mahmassani (Talebpour and Mahmassani, 2016a) investigated string stability in a platoon of connected and automated vehicles and showed that automation has a more significant effect on stability than connectivity.

In Talebpour et al. (Talebpour et al., 2017), critical concepts have been tackled to ensure connectivity in a mixed driving environment with connected and automated vehicles. The authors showed how the correlation between communication range and connected vehicle density affects the connectivity level and the string stability of the traffic flow. Arefizadeh and Talebpour (Arefizadeh and Talebpour, 2018) proposed a new constant time headway strategy for automated vehicle platoon to prevent shockwave formation caused by speed drops.

Lane changing maneuver

As one of the critical driving maneuvers, many approaches have been suggested to generate lane-changing trajectories safely and reliably. Typically, adopting a function that describes the geometric representations of trajectory has been a dominant approach. Various functions have been used in the literature (see Table 1 for more details). In this study, to compare several lane-changing trajectory functions, several criteria, including continuity and smoothness of the curvature, minimal jerk, minimal length, and minimal curvature, have been considered.

**Circular arcs:** This is a simple approach to describe a lane-changing trajectory. By putting circular arcs together, a simple trajectory can be obtained, and the radius of curvature is determined by lateral and longitudinal displacement after a lane-changing maneuver. Accordingly, based on the constant curvature, the lateral position can be determined depending on the velocity and the corresponding longitudinal position. However, this method has certain disadvantages. For instance, additional consideration is required to deal with the discontinuity issue that happens from connecting two arcs. More importantly, speed adjustment during the lane-changing maneuver can become very complicated.

**Polynomial:** This is the most common approach to represent a lane-changing trajectory. The trajectory can be generated by considering position constraints that the vehicle has to pass through. The key advantages of this approach are that these curves are easy to compute and can always generate a continuous curvature. Moreover, the polynomial can be of any order and being able to consider higher-order polynomials (polynomials higher than 4 degrees) givesmore flexibility in terms of designing the trajectory. Normally, using higher order allows to increase flexibility in terms of trajectory shaping. In Cong et al. (Cong et al., 2010), 4th order polynomial was used and constraints like maximum acceleration and maximum driving force were considered. In another study, Nelson and Winston (Nelson, 1989) used 5th order polynomial to describe the lateral position of the vehicle.

|  |  |
| --- | --- |
| Table . Lane-changing trajectory generation methods. | |
| **Methods** | **Description** |
| Circular Arc | Interpolation of waypoints using a circle |
| Polynomial | Interpolation of waypoints using polynomial curves |
| Bezier curve | Generating curves based on selected the Bezier control points |
| Spline | Generating a piecewise trajectory |

**Bezier curves:** Originally, the Bezier curve was used in the field of computer graphics to obtain the curves. Bezier curve is based on a concept called *control points*. Rastelli et al. (Rastelli et al., 2014) showed that it is possible to shape the trajectory curve depending on the convex hull based on the control points. Therefore, this approach is a very intuitive way to manipulate the trajectory and it has low computational cost to generate. Gonzalez et al. (González et al., 2014) obtained the lane-changing trajectory using the Bezier curve considering the roadway constraints.

**Spline curves:** In a nutshell, this method is a piecewise polynomial curve. After generating points, each sub-interval between points can be described using any type of curves, including polynomial and circular ((Piazzi et al., 2002) and (Bacha et al., 2008)).

**Occupancy grid:** By discretizing the search space into a grid (as shown in Figure 1.a) and checking the possibility that a grid is occupied by an obstacle, a feasible trajectory can be generated. Out of many graph-based approaches, this method is relatively straightforward to implement and computationally efficient. In Schroder et al. (Schroder et al., 2008) and Kolski et al. (Kolski et al., 2006), for each cell, the possibility of being occupied by an obstacle and the corresponding risk was calculated to check the feasibility of a trajectory.

**State Lattice:** This approach also starts from the reconstruction of the environment but into a different search space, called state lattice as shown in Figure 1.b. Pivtoraiko et al., (Pivtoraiko and Kelly, 2005) and Pivtoraiko et al. (Pivtoraiko et al., 2009) generated the trajectory based on set lattices that guarantee the feasibility of the path.

In this study, the polynomial-based lane-changing trajectory generation method is adopted. Several factors contributed to this selection. First, the approach has a low computation overhead and can use the boundary conditions to generate the polynomial that best describes the optimal trajectory. Second, by adding the 6th order term, we can optimize the trajectory curvature to minimize the impact on the target lane. These features provide the trajectory optimization process with the opportunity to predict the behavior of the MPC and to minimize the disturbance caused by the lane-changing maneuver.

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Figure .Illustration. (a) Occupancy grid, and (b) State lattice.

Formal problem statement

The main focus of this section is on co-optimizing the lane-changing maneuver and car-following behavior of the vehicle directly impacted by the lane-changing maneuver considering all the proceeding vehicles in the platoon. Figure 2 illustrates this problem. Figure 3 presents the situation before the lane-changing maneuver. The lane-changing can happen between any feasible gaps throughout the platoon. Figure 3 presents the situation after the lane-changing, where the vehicle that is impacted by this maneuver is called "platoon head". Note that even though we keep this terminology in the remainder of this study, it doesn't mean that the platoon will break into two sub-platoons after the lane-changing maneuver. However, our focus is only on the vehicles in this subset of the platoon that will be impacted directly or indirectly by the lane-changing maneuver.

A close up of a device

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Figure . Illustration. Problem statement: Before lane-changing maneuver.

A screenshot of a cell phone

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Figure . Illustration. Problem statement: After lane-changing maneuver.

Model Development

Car-following controller

To capture the full impact of the lane-changing maneuver on the target lane, we consider all the vehicles that can be potentially impacted by this maneuver. In Figure 3, blue vehicles constitute a platoon following the platoon head, SV. SV directly responds to the lane-changing vehicle (LV, the black vehicle in Figure 3), and its acceleration is controlled based on the MPC. The remaining vehicles in the platoon can be either automated vehicles or human-driven vehicles. The control law of the automated vehicles in the platoon is based on a well-established platooning strategy that guarantees string stability. The human-driven vehicles are assumed to follow the Intelligent Driver Model(IDM) (Treiber et al., 2000). The details of the platooning strategy and MPC are presented below.

Platoon members: Constant Time Headway policy

In this study, we adopt a constant time headway (CTH) policy to control the automated vehicles in the platoon (except for SV that is controlled based on the MPC). According to the CTH setup, vehicle acceleration is selected based on safety criteria and desired time headway of the platoon. In Figure 3, the spacing error between the predecessor (at position ) and its follower (at position ) is determined based on the velocity of follower, , the vehicle length, , and the desired time headway,:

Figure . Equation. Spacing error between the predecessor and its follower.

According to Swaroop (Swaroop, 1997), the acceleration to make the spacing error converge to zero asymptotically (i.e., string stable platoon) is given by:

Figure . Equation. Acceleration of a string stable platoon.

where should be greater than zero to ensure string stability. However, Swaroop's controller can be inefficient considering situations where a platoon is disturbed by LV. Even though the control law of the equation in Figure 5 can eventually and gradually control the shockwave resulted from a lane-changing maneuver, this controller can create large jerk values right after the lane-changing maneuver. This is mainly due to the sudden change in . More details about the behavior of this controller under such scenarios are presented in the section for results and discussion.

To address this issue and to ensure passenger comfort while accepting LV as a new predecessor, we need to design a car-following controller that can be used for SV considering a sudden appearance of LV.

Platoon head: Model Predictive Controller approach

Let’s consider the following scenario where SV follows a different control law than the rest of the platoon. Using the velocity of SV, , the velocity of LV, , and the longitudinal distance between them, , we can describe the longitudinal dynamics as:

Figure . Equations. Longitudinal dynamics equations.

where is the sampling time. Furthermore, to integrate the longitudinal dynamics into the control design process using MPC, we discretize the differential equations in Figure 6 using the Euler method, which results in:

Figure . Equations. Discretized longitudinal dynamics equations.

In Figure 8, SV needs to adjust its speed to have a desired and safe distance to LV. Accordingly, CTH is chosen as a spacing strategy, where the desired distance is proportional to the velocity of SV, and MPC outputs acceleration/deceleration values of SV for spacing two vehicles based on the prediction of LV's position and speed. However, MPC works based on the transmitted acceleration profile of LV for the lane-changing maneuver. Based on the acceleration profile that LV will take, MPC predicts how LV will behave. So, MPC predicts the future using the actual and then determines how will change based on the . Accordingly, by predicting the future states of LV, , and defining a cost function to determine the desired future states, , MPC can generate an optimal control input sequence, . Those optimized values will be control inputs to SV to adjust according to the CTH strategy under the penalty on the rapid change of acceleration, .

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Figure . Illustration. Variables for vehicle platoon control using MPC.

We define two cost functions by decoupling the reaction scenario into two parts: (1) SV needs to slow down because is shorter than the predetermined minimum time headway 1 second, and (2) SV recovers its desired velocity (i.e., velocity before reacting to LV) when LV becomes faster than SV. Note that the latter ensures that the distance does not increase indefinitely. Two cost functions are introduced.

***First cost function: Keeping a safe distance***

SV decelerates by penalizing the tracking error based on and :

Figure . Equation. Difference between desired time headway of SV and LV.

where represents the desired time headway. MPC tries to minimize the difference between and . However, considering the safety distance according to Figure 9 can result in unrealistic/aggressive acceleration/deceleration choices and the MPC cannot adapt to the changes in the target lane. SV needs to consider how LV behaves in the target lane (e.g., whether LV accelerates or decelerated while adapting to the traffic flow dynamics in the target lane). Therefore, SV should check the acceleration of LV and also penalizes the difference between the acceleration of SV and LV. Note that, however, the following term is only required when the distance is sufficient.

Figure . Equation. Difference between acceleration of SV and LV.

This makes car-following behavior more reliable by depending not only on the distance itself but also on how the distance changes implicitly. In addition to equations in Figure 9 and Figure 10, a third term is necessary to prevent rapid changes in the acceleration of SV. Therefore, the cost function for making the safe distance under the consideration of LV's acceleration change is given by:

Figure . Equation. Cost function for making the safe distance.

Accordingly, SV tries to have the minimum time headway according to the cost function of Figure 11 until it reaches LV's velocity.

***Second cost function: Speed recovery***

Speed recovery is a critical factor for SV that the first cost function cannot achieve. Accordingly, we need a cost function to ensure that SV can recover its velocity after the initial deceleration to create a gap. In this case, LV is faster, and increases without any upper bound. Therefore, we replace the equation in Figure 9 with the following term:

Figure . Equation. Difference between desired speed of SV and LV.

Where is the desired speed. Based on this equation, by starting from a smaller velocity than LV and slowly increasing the speed, SV recovers its velocity very smoothly in a safe way. We propose the following cost function to be used for speed recovery:

Figure . Equation. Cost function for speed recovery.

Note that an additional terms included in this cost function to prevent the large value for the acceleration. In Figure 11, this term does not exist to avoid the case where SV tries to stop or slowly slow down because of the penalty from the large deceleration even though it still needs to keep slowing down. However, by having the fourth term in Figure 13, MPC starts to put relatively more emphasis on driving comfort once the distance stops decreasing.

The above cost functions are considered in conjunction with the following constraints:

|  |  |  |
| --- | --- | --- |
| Subject to: |  |  |
|  |  |  |
|  |  |  |

Figure . Equation. Constraints for the safe recovery problem.

Where is the state vector and is the prediction horizon. When the first cost function is activated, MPC works based on a short prediction horizon. It is to address the need for a quick adaptation to LV’s behavior during a short period until the second cost function starts to kick in. We use the longer prediction horizon for the second cost function. In the first equation of Figure 14, longitudinal dynamics are described and predicted using the state vector , and the control input . Acceleration changes are limited according to the second equation in Figure 14. As one of the important aspects of the MPC design, we need to determine the weights in the cost functions. We have to consider if different values have to be used for weights depending on the scenario. In detail, the weight in Figure 11 plays an important role in how naturalistic the car-following behavior is and has to change according to how much of emphasis the cost function has to put on adjusting to LV’s acceleration changes. Since the deceleration to make minimum safety distance is based on both position errors, , and how those position errors change by looking at the acceleration of LV, , tuning of these two weights ( and ) determines the trade-off between both effects. Therefore, either when the distance is relatively sufficient in the perspective of SV or when LV is not much slower than SV, the cost function includes the term in Figure 10 to handle not just safety but also naturalistic driving behavior. Therefore, by using different values for , we design MPC for various scenarios, and MPC can show smoother car-following behavior in response to LV. Note that SV uses the designed MPC and Swaroop's controller and IDM are used for other followers in the same platoon since they respond to a predecessor in the same platoon.

The following parameter values and constraints are used along with Figure 11 and Figure 13 in this study:

* Sampling time: = 0.2 sec
* Prediction horizon: = 2 for Figure 11 and = 10 for Figure 13
* Constraints on acceleration change per : 2.52.5.

Lane-changing trajectory

There are many ways to generate the lane-changing trajectory as introduced earlier, and among them, we choose an efficient, yet effective approach based on a high-order polynomial.

Trajectory generation: 6th order Polynomial

Papadimitriou et al. (Papadimitriou and Tomizuka, 2003) suggested trajectory generation based on two fifth-order polynomials for X-axis and Y-axis. Furthermore, by adding the sixth-order term to the polynomial for the X-axis, they adjusted the trajectory for collision avoidance. In this study, we use the fact that the sixth order term is adjusting the curvature of the lane-changing trajectory. Note that in this research, the purpose is not to avoid objects but to minimize the impact of the lane-changing maneuver. Accordingly, following the approach in Papadimitriou et al. (Papadimitriou and Tomizuka, 2003), we first fix a particular lane-changing maneuver duration and can define the following equations:

Figure . Formulas. Trajectory curve for the lane-changing maneuver.

These equations satisfy both the initial boundary condition () and the final boundary condition (). Therefore, by changing the sixth order coefficient, in Figure 15, the trajectory can slightly curve, and consequently, the lane-changing vehicle can keep a minimum distance with a particular moving object under dynamic constraints such as maximum longitudinal and lateral acceleration values. Figure 16 illustrates the impact of on the trajectory shape. In this figure, trajectories are generated for the same boundary conditions and fixed duration, . However, because of the different value for , trajectories are different in terms of the curvature, and we expect each trajectory to have a different influence on the target lane while still having a smooth polynomial-based trajectory. In this study, utilizing this methodology for trajectory generation, we introduce an optimization system to minimize the disturbance in the target lane. That is to prevent any hard braking of followers in the target lane that can occur due to the lane-changing maneuver.

A close up of a map

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Figure . Diagram. Different curvatures of the lane-changing trajectory depending on .

Trajectory optimization

The overall schematic for identifying the optimum trajectory is depicted in Figure 17. For the two vehicles (Red and Blue) in the target lane, the lane-changing vehicle (Black) evaluates two sets of possible trajectories for final boundary conditions considering the desired time headway after the lane-changing maneuver and the desired maximum lateral acceleration. The time headway can be chosen for individual vehicles or the platooning strategy. Note that lane-changing duration, , can directly impact the lateral acceleration in Y-axis. The minimum can be calculated based on the maximum lateral and longitudinal accelerations (identified based on the two polynomials that represent a location in X and Y axes). However, it is easy to show that larger values correspond to smaller fluctuations in the target lane.

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Figure . Illustration. Sets of feasible trajectories depending on the gap choice.

Consequently, the largest possible value for (that does not violate the maximum acceleration values and meets the expectation in the real-world setting) is desirable. Therefore, we define the minimum considering the maximum available lateral acceleration . Assuming that at two boundary conditions, and are all zero, small values for the maximum allowable lead to longer . Therefore, we define the maximum value for , and then we obtain the minimum value of . Moreover, we also noticed that bigger the is, the smaller speed fluctuation in axis has because is also based on the polynomial. Therefore, once we obtain the minimum value, it is better to have a bigger value for . In this research, to ensure the feasibility of the lane-changing maneuver in terms of lateral acceleration, we fix the lane-changing duration to six seconds, which is considered reasonable also based on the findings of Toledo et al. (Toledo and Zohar, 2007). The following optimization system is formulated to identify the best lane-changing trajectory:

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| Subject to: |  |  |
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Figure . Formulas. Optimization problem for finding the best lane-changing trajectory.

where and indicate the maximum acceleration in and directions, respectively. denotes the total deceleration effort of SV in response to the lane-changing trajectory calculated as illustrated in Figure 19.

A close up of a map

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**Source: FHWA**

Figure . Illustration. Trajectory optimization by minimizing the total control effort, S.

Based on this figure,

Figure . Equation. Total control effort.

where is when SV starts to decelerate in response to LV and is when SV starts the speed recovery process with positive acceleration. Figure 21 illustrates the process of calculating . The key aspect of this optimization is to calculate . Accordingly, after generating a trajectory for a given set of available gaps, LV predicts the acceleration profile of SV according to the designed MPC. Note that it is assumed that LV has information about the MPC design (or at least the MPC acceleration predictions) through vehicle-to-vehicle (V2V) communications. LV first generates a set of feasible value of only under the consideration of physical constraint (i.e., lateral acceleration and longitudinal acceleration) as in Figure 21. In this research, the constraint on the maximum longitudinal acceleration can only affect the range of feasible . Then, we can get multiple sets of possible trajectories by considering each vehicle in the target lane as the future leader. From there, LV chooses optimal trajectory based on the effect of each trajectory on the target lane calculated using the objective function (Figure 18 and Figure 20).

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**Source: FHWA**

Figure . Flowchart. Strategy for the accurate prediction of MPC behavior.

The key element in this optimization system is the methodology to predict the states of each vehicle since each lane-changing trajectory has a different impact on the target lane and imposes a unique disturbance to the system. Accordingly, we propose a methodology to evaluate the impact of each trajectory on the changes in vehicles' speed in the target lane; thus, capturing the impact of lane-changing on total travel time. After generating a trajectory for a given set of available gaps, the model predicts the acceleration profile of the platoon head according to the designed MPC Note that it is assumed that the lane-changing vehicle has information about the MPC design.

The optimization will be based on the prediction of the acceleration set generated by cost functions (Figure 11 and Figure 13). Therefore, the strategy for the optimization can be described as in Figure 21. First, the lane-changing model considers trajectory candidates based on physical constraints. For example, in Figure 17 the lane-changing vehicle (black) can obtain two sets of trajectory candidates for two available gaps into which it can move. Consider representing the sequence of acceleration of the platoon head while adapting to the lane change car with the sixth order trajectory coefficient . Here, represents the duration required to adjust to LV (time required to get back to the desired velocity after the slowdown due to the lane-changing maneuver). The important factor here in this approach is that the model does not compute the entire acceleration sequence of the platoon head at once using a long prediction horizon, which is what a generic MPC does. The prediction done that way can be inaccurate, and this is regardless of if the utilized prediction horizon is long or not. Instead, we first assume that V2V between the platoon head and the lane change car is available, and MPC modeling of the platoon head is known to the lane change car. Consequently, from the perspective of LV, it is possible to predict the acceleration of SV, , and the updated state of SV by applying . From there, LV does the same process iteratively and predict SV's state assuming that the MPC of SV works in the same way just based on a new prediction about LV at . LV continues this process until , when SV is predicted to stop decreasing its velocity. This is possible only because of the fact that LV knows both the MPC structure and the trajectory candidates in advance. Therefore, LV handles accuracy issues by implementing an iterative prediction approach as in Figure 21. Furthermore, LV can figure the optimal trajectory out of trajectory candidates and minimizes the shockwave and the travel time of the platoon. In other words, in Figure 19, the model computes the area, which represents the entire control efforts required for the platoon head until it stops decelerating. Accordingly, we try to get the value that minimizes the equation in Figure 20 and use it as 6th order coefficient of the polynomial trajectory.

Gap generation

In addition to designing and MPC controller to mitigate the negative impacts of lane-changing at the merge locations, one can generate enough gap for the lane-changing vehicle by introducing a forward-moving shockwave. Before controlling cars ahead of a lane change vehicle (LV), we have to determine a set of time headway () values that the vehicles have to obtain. Time headway values will be smaller than or equal to the original time headway () that platoon members follow before the system starts to work. Therefore, we can describe the change of relative position of platoon members in the platoon as in Figure 22. In this figure, circles represent vehicles, and we compare two platoons. The platoon above is when platoon members keep their constant , and the other one below is when some of the platoon members (green circles) reduce their . In other words, the system makes vehicles move forward relative to their original position in a platoon. Green circles are cars controlled to achieve their new , which is smaller than or equal to , and we can see that their resulting position are relatively ahead of their assumed position with the constant time headway, . Consequently, LV is assigned a space to join in the target lane as a new member of the platoon. Automatically, LV will have less impact on the vehicle following it.

In this study, we consider two formulations to generate distribution: one approach is to maximize the generated gap, and the other approach is to provide LV the required gap based on the final position of the LV at the end of the lane-changing maneuver. Note that it is assumed that the final position of the LV will be communicated to the vehicles.

A picture containing drawing, meter

Description automatically generated

**Source: FHWA**

Figure . Illustration. Time headway adjustment: Relative position comparison: No Dynamic MPC (up) vs. Dynamic MPC (down).

Note: Circles filled by green represent that they are controlled by DMPC. We can see that by decreasing h of DMPC vehicles (→), vehicles' position.

Case1: Maximizing Distance

This is to maximize a new space given the particular number of vehicles, , to control and the constraint on their distribution. The strategy to distribute time headways of all vehicles are based on the time headway variances. The following optimization is developed:

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| --- | --- | --- |
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| Subject to: |  |  |
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Figure . Formulas. Optimization problem for case 1: maximizing distance.

where denotes the headway of the th vehicle. In detail, checking the is based on the idea that a large variance of vehicles' speed can result in shockwave formation and unsafe driving instances. Therefore, by having a constraint on the variance of , we try to obtain the set of s that does not cause a variance larger than a particular number. As long as a variance of a set does not exceed the safe limit, s should be acceptable.

Case2: New Gap to make is determined

This case assumes that LV defines the size of the gap it needs in advance. In such a case, all the vehicles can be controlled focusing on making the gap just larger than a particular size instead of maximizing it. Here, we assume that the platoon can receive the information on the expected gap from LV before the execution of the lane-changing maneuver. The following optimization is formulated:

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| Subject to: |  |  |
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Figure . Formulas. Optimization problem for case 2: new gap to make is determined.

Chapter 3. MODEL CALIBRATION AND VALIATION

To calibrate and validate the joint application calibration developed under CAV-AMS Phase II project, two key features have been developed and integrated into Northwestern’s microscopic simulation platform:

* Speed Harmonization: A set of novel speed harmonization algorithms were developed that utilize machine learning to predict the onset of congestion and to activate the speed harmonization in a highway segment. These algorithms also utilize various methods of communicating the updated speed limits to the connected vehicles (automated or human-driven) and non-connected vehicles (automated or human-driven).
* Merge coordination: A couple of algorithms were developed to enable merge coordination in connected and non-connected driving environments. These algorithms aim to prevent shockwave formation in the target lane, even at very small time-headways.

In addition to these models, the simulation platform utilizes several already calibrated and validated car-following and lane-changing models for non-connected human-driven vehicles, connected human-driven vehicles, connected automated vehicles, and non-connected automated vehicles (Talebpour and Mahmassani, 2016a).

Since most of the utilized models were already calibrated and validated based on the NGSIM US-101 dataset (2007), the focus of this calibration and validation effort will be on the calibration of car-following models of human-driven vehicles to capture the effects of interacting with automated vehicles on driver behavior. The validation effort will ensure the accuracy of the calibration process.

Dataset

Vehicle trajectories are one of the cornerstones of modern traffic flow theory with applications in driver behavior studies and automated vehicle research. Unfortunately, the existing vehicle trajectory datasets are limited, mostly due to the high cost of data collection and preparation. Moreover, with the arrival of advanced driver assistance systems (ADAS) and automated vehicles, there is a potential to see changes in human driving behavior when interacting with these technologies. As a result, there is a need for new vehicle trajectory datasets that cover various levels of automation. Aerial imagery using small unmanned aerial vehicles (UAVs) is an economical and effective solution to collect trajectory data.

To address the shortcomings of the existing vehicle trajectory datasets, a new trajectory dataset was collected on Interstate 35 in Austin, TX (See Figure 25). A platoon of three SAE Level 1 automated vehicles with ACC technology was circulating in the traffic stream during the data collection. Two UAVs (e.g., drones) were used for the aerial videography of the traffic stream. The trajectory of the vehicles can be extracted from the video frames recorded in the bird's-eye view from a segment of the roadway (See Figure 26). In every video frame, the location of the vehicles can be estimated for a fixed coordinate system and reference point on the ground. Every video recording is converted to a sequence of images (i.e., frames) separated at a constant rate over time (e.g., 25 frames per second). Tracking the location of any vehicle over the sequence of images enables extracting the vehicle's trajectory over time.

The vehicle trajectory extraction is performed in four steps: image stabilization, vehicle detection, vehicle tracking, and trajectory construction. In the image stabilization step, all the images are transformed to match a reference field of view. Then the vehicles are detected in every frame and tracked over the sequence of images. Finally, the vehicles' location and trajectories are constructed by converting the image coordinates to the adopted reference coordinates on the ground. Figure 27 shows a sample of collected vehicle trajectory data.

A close up of a map

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**Original Photo: © 2019 Google® (See Acknowledgements).**

Figure . Photo. Data collection location on Interstate 35 near Austin, TX.

|  |
| --- |
| **A screen shot of a computer  Description automatically generated(a) Bird’s Eye View** |
| **A close up of a computer  Description automatically generated** |
| 1. **Vehicle Detection Using Convolutional Neural Network** |
| **A picture containing road, car, green, electronics  Description automatically generated** |
| 1. **Vehicle Tracking**   **Source: FHWA**  Figure . Photo. Vehicle detection and tracking in aerial images.  A close up of a curtain  Description automatically generated  **Source: FHWA** |

Figure . Illustration. Sample trajectory data collected on Interstate 35 near Austin, TX.

Calibration Approach

This study adopts the genetic algorithm calibration approach introduced by Hamdar et al. (Hamdar et al., 2009). The approach relies on comparing the driving behavior in the dataset with the simulated behavior based on a set of model parameters. For car-following models, the error will be calculated based on the error in the gap between the lead vehicle and the target vehicle:

Figure . Equation. Error in the gap between the lead vehicle and the target vehicle.

Where , , and . For lane-changing models, the same process is followed with one key difference; that is the error in the gap between the lane-changing vehicle and both new leader and new follower is considered.

Once the error function is defined, the genetic algorithm heuristic can be implemented as follows:

* The parameters of a car-following/lane-changing model are initialized to random numbers. Each set of these parameters is called a “chromosome” and the total of chromosomes will be created.
* The “fitness” of each chromosome is determined using the aforementioned error function.
* Except for the chromosome with the lowest error value, every other chromosome will be evolved through cross-over and mutation (see Hamdar et al. (Hamdar et al., 2009) for the definition of cross-over and mutation in the genetic algorithm).
* The process is terminated once a minimum error threshold is achieved by the best chromosome. The parameters of that chromosome will form the calibration results.

Following the procedure outlined above, an initial set of 100 parents will be initiated. These parents will produce 900 children at each iteration and the top 99 children will join the best of the parents to move to the next iteration. The calibration process stops once the error is below 5% or less than 0.1% improvement in error is observed for more than 20 consecutive iterations.

Calibration and Validation Process

The behavioral parameters of drivers in microscopic simulation models are expected to be correlated. Kim and Mahmassani (Kim and Mahmassani, 2011) presented a methodology to capture this correlation across the parameters of each driver. They showed that sampling from the empirical data while accounting for the correlation between the parameters of each sample (individual drivers) is the best method for capturing heterogeneity in microscopic simulation models. This study will utilize the same method for the calibration of car-following and lane-changing models.

To calibrate and validate the model, each vehicle trajectory in the dataset will be divided into calibration and validation sets. The calibration set will have about four times more data points than the validation set, all selected randomly from the data points in the vehicle trajectory dataset. Utilizing the same error function presented in the previous section, the model will be first calibrated using the data in the calibration set. The calibrated model parameters will be then used to simulate the data in the validation set and the results (the gap between vehicles) will be compared. The outcome of this calibration and validation process is a set of car-following/lane-changing parameters for each vehicle trajectory in the dataset.

As discussed previously, the models utilized in Northwestern’s simulation platform have gone through a similar calibration and validation process based on the NGSIM US-101 dataset. Accordingly, this study will only focus on the cases when a human driver interacts with an automated vehicle in the dataset. The focus will be on a human driver following an automated vehicle. Note that Rahmati et al. (Rahmati et al., 2019) showed that there is potential to see significant changes in driver behavior in this case and real-world data will be utilized to quantify these changes.

A note on calibrating and validating car-following and lane-changing models for automated vehicles

Regarding car-following models of automated vehicles, Northwestern’s simulation framework utilizes the ACC/CACC models developed by Milanés and Shladover (Milanés and Shladover, 2014). These models were calibrated based on empirical data. Accordingly, the car-following behavior of automated vehicles will not be calibrated again in this study.

Regarding lane-changing models of automated vehicles, these models are designed based on the capabilities and characteristics of Texas A&M’s automated Chevy Bolt EV. Accordingly, any lane-changing trajectory generated by the models can be followed in the real-world.

**Calibration and Validation Results**

Table 2 shows the calibration results along with the ANOVA test outcome for the Austin data along with the data collected by Rahmati et al. (Rahmati et al., 2019). Two of the key model parameters show no statistically significant difference (i.e., and ), indicating that drivers’ utility in response to acceleration and deceleration were the same for both cases of following an AV and following another human driver.

Figure 29 shows the function for the average values in scenarios A and B. Note that since the average and are almost the same value, functions are almost identical. This figure indicates that drivers were less sensitive to deceleration and they did not feel significant disutility from braking regardless of the deceleration value. In a real-world context, however, drivers seek to travel at their desired speed and minimize their travel time, while avoiding crashes. Therefore, they try to minimize hard braking and favor higher acceleration rates (considering the comfort level). However, in the context of the experiments conducted in this study, drivers did not feel any urgency to reach the end of the test (especially since each test took about 2-3 minutes to finish). Their main focus was on safely following their leader assuming they are driving in a highway environment. Accordingly, the result (as presented in Figure 29) was expected.

Table . Prospect theory model calibration results.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Parameters** | **Human Following**  **(Mean)** | **Human Following**  **(Standard Deviation)** | **AV Following**  **(Mean)** | **AV Following**  **(Standard Deviation)** | **ANOVA Test**  **(*p*-value)** |
|  | 0.26 | 0.44 | 0.27 | 0.37 | 0.3597 |
|  | 112200.00 | 20564.31 | 82377.78 | 20360.71 | 0.0001 |
|  | 0.68 | 0.66 | 0.68 | 0.64 | 0.9402 |
|  | 4.98 | 2.42 | 5.33 | 2.61 | 0.4707 |

Unlike function parameters, the crash weighing factor, , showed a statistically significant difference between the two sets of experiments. Following an AV, human drivers’ behavior resulted in much less compared with the case of following another human driver ( is 36% less for the AV following case than the human following case). Such a significant difference shows that human drivers are more comfortable following the AV compared with another human driver and they feel safer. Such an observation on can also be interpreted from the risk-taking perspective. The smooth behavior of the AV encourages more risk-taking behaviors by the following human driver, resulting in lower values of . Finally, is slightly higher for the AV following scenario, indicating more stable driving behavior by the follower. Note that the difference in between Scenario A and B is not statistically significant.

Figure 30 translates the subjective utility functions, , into acceleration probability density functions, for various values of relative speed. Accordingly, three cases are illustrated, 1) follower is approaching the leader with a relative velocity of +10 mph (green plots), 2) keeping a constant space headway with a relative velocity of 0 mph (orange plots), and 3) increasing the space headway with the leader with a relative velocity of -10 mph (blue plots). Comparing the two scenarios, the model calibrated based on the AV following results in more stable behavior, while the model calibrated based on the human following results in higher deceleration rates. Overall, the findings of model calibration indicated a clear difference between humans’ AV following and human following behavior.

|  |  |
| --- | --- |
|  |  |
| **(a)** | **(b)** |

Source: FHWA

Figure . Diagrams. as a function of acceleration (a) Scenario A, and (b) Scenario B.

|  |  |
| --- | --- |
|  |  |
| **(a-1)** | **(a-2)** |
|  |  |
| **(b-1)** | **(b-2)** |
|  |  |

Source: FHWA

Figure Diagrams. Utility and probability density function for human following and AV following.

Note: Utility (a-1 and a-2) and probability density function (b-1 and b-2) for human following (a-1 and b-1) and AV following (a-2 and b-2) for V = 10mph, s = 10m, and various values of ΔV (ΔV = -10mph is represented by blue line, V = 0mph is represented by orange line, and V = 10mph is represented by green line). All model parameters are selected based on the calibration results.

Chapter 4. BASIC GUIDANCE ONE MODEL IMPLEMENTATION

Figure 31 illustrates the flowchart for the implementation of the merge coordination algorithm. The integration with any microscopic traffic simulation is possible. The model input is vehicle location, speed, and acceleration (both merging vehicles and other vehicles in the target lane). The output, however, can be different based on the capabilities of the microscopic simulation tool. Since the model operates in a 2D environment (i.e., both lateral and longitudinal movements are considered), the implementation in most commercially available microscopic simulation tools requires an adjustment to the model outcome. In other words, instead of outputting both lateral and longitudinal movements, the model should be adjusted to output longitudinal location and lane number.

In addition to the implementation of the lane-changing maneuver, the existing implementation is capable of outputting the impact on shockwave formation. This part can be removed, if found to be unnecessary for the study, without impacting the merging/lane-changing algorithm.

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**Source: FHWA**

Figure . Flowchart. Implementation of merge coordination system.

Chapter 5. SENSITIVITY STUDY

Implementation of the developed model into a traffic simulation tool

The merge coordination system was implemented in Northwestern University’s microscopic simulation tool. The framework was incorporated into a microsimulation tool developed in Python. The microsimulation tool is a special-purpose platform for simulating mixed traffic conditions on freeways with the possibility of including connected vehicles and autonomous vehicles in the system.

In the simulation platform, distinct car-following models are defined to specify the behavior of each agent: 1) Manually driven vehicles (regular vehicles); 2) Connected vehicles; and 3) Automated vehicles.

In the microsimulation platform, manually driven vehicles use the acceleration model first developed by Hamdar et al. (Hamdar et al., 2008) and extended by Talebpour et al. (Talebpour et al., 2011). The model was formulated based on Kahneman and Tversky’s prospect theory. Two value functions, one for modeling driver behavior in congested regimes and the other for modeling driver behavior in uncongested regimes, were introduced. The following formula shows the value function for the uncongested regime:



Figure . Formula. Value function for the uncongested regime.

Where denotes the value function for the uncongested traffic conditions. and are parameters to be estimated and calibrated and is used to normalize the acceleration. On the other hand, the following formula shows the value function for the congested regime:



Figure . Formula. Value function for the congested regime.

Where denotes the value function for the congested traffic conditions. and are parameters to be estimated and calibrated. At each evaluation time step, the driver evaluates the gain from a candidate acceleration selected from a feasible set of values. The surrounding traffic condition is taken into consideration by the driver throughout the acceleration evaluation process. The driver utilizes the following binary probabilistic regime selection model to evaluate each acceleration value:



Figure . Formula. Binary probabilistic regime selection model.

Where , , and denote the expected value function, the probabilities of driving in a congested traffic condition, and the probability of driving in uncongested traffic conditions, respectively. After calculating the expected value function, the total utility function for acceleration could be written as follows:



Figure . Formula. Total utility function for the choice of acceleration.

Where is the crash probability. Finally, the following probability density function is used to evaluate the stochastic response of the drivers:

 

Figure . Formula. Probability density function for the evaluation of drivers’ stochastic response.

Where is the sensitivity of choice to the utility .

Connected vehicles are capable of exchanging information with other vehicles and infrastructure-based equipment. The information is exchanges through the vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications networks. As a result, the driver receives information about other connected vehicles as well as updated information containing TMC decisions (e.g., real-time changes in speed limit). The drivers’ behavior may change based on the information conveyed to the driver. The reliability and the frequency of the information received by the driver plays a significant role in the drivers’ behavior and on the overall performance of the traffic network.

An active V2V communication network allows the drivers to be aware of other drivers’ behavior, the driving environment, road condition, and weather condition. As a result, the driving behavior could be modeled using a deterministic acceleration modeling framework. The simulation tool utilizes the Intelligent Driver Model (IDM) to model this connected environment. Because the IDM can capture various congestion dynamics and provides greater realism than most of the deterministic acceleration modeling frameworks.

The acceleration model specified by the IDM entails the vehicle’s current speed, the ratio of the current spacing to the desired spacing, the difference between the leading and the following vehicles’ velocities, and subjective parameters such as desired acceleration, desired gap size, and comfortable deceleration.

Figure . Formula. The intelligent driver acceleration model.

Where is the free acceleration exponent; is the desired time gap; is the maximum acceleration; is the desired deceleration; is the jam distance; and is the desired speed. These parameters need to be calibrated to better capture the behavior of connected vehicles.

If the V2V communication network is inactive, the driving behavior of connected vehicles would be similar to that of isolated-manually driven vehicles. In the presence of V2I communications, the TMC decisions, such as the speed limits in the case of speed harmonization, could be transferred to the drivers. However, their reaction times would still be like regular drivers.

Automated vehicles can continuously monitor other vehicles in their vicinity, which results in a deterministic behavior in interacting with other drivers. Furthermore, they can quickly react to any perturbations in the driving environment. Therefore, the car-following behavior of automated vehicles could be specified by a deterministic modeling framework. Talebpour and Mahmassani (Talebpour and Mahmassani, 2016b) developed a car-following model for automated vehicles based on the previous simulation studies by Van Arem et al. (Van Arem et al., 2006) and Reece and Shafer (Reece et al., 1993). They simulated similar individual sensors installed on the automated vehicles to generate the input data for the acceleration model.

Considering the sensor range and limitations in accuracy, the automated vehicles must be ready to react to any situation outside of their sensing range once it is detected (e.g., a vehicle at a complete stop right outside of the sensors detection range). Furthermore, if a leader is spotted, the speed of the automated vehicle should be adjusted in a way that allows it to stop if the leader decides to decelerate with its maximum deceleration rate and reach a full stop. Considering different situations that require the immediate reaction of the automated vehicle, the maximum safe speed can be calculated using the following equations:

Figure . Formula. Maximum speed of automated vehicles.

Where n and n-1 represent the automated vehicle and its leader, respectively; is the position of vehicle n; is the length of vehicle n; is the speed of vehicle n; is the reaction time of vehicle n; and is the maximum deceleration of vehicle n.

Besides the safety constraint, the following formula, adopted from the model proposed by Van Arem et al. (Van Arem et al., 2006), updates the acceleration of the automated vehicle at every decision point:

Figure 39. Formula. Acceleration model for automated vehicles.

Where is the acceleration of vehicle n; and , , and are model parameters that need to be calibrated. is the spacing and is the maximum between the minimum distance (), following distance based on the reaction time (), and safe following distance (). In this study, the minimum distance is set at 2.0 m and is calculated according to the following formula:

Figure . Formula. Safe following distance formula.

Finally, the acceleration of the automated vehicle can be calculated using the following equation:

Figure . Formula. Acceleration of automated vehicles.

Where is a model parameter. Van Arem et al. (Van Arem et al., 2006) suggested using the following values for the model parameters:1, , , and

Design of simulation experiments AND Simulation results

In the following subsections, we evaluate the performance of the proposed joint optimization of lane-changing trajectory, car-following controller, and gap generation mechanisms.

Car-following control

We simulate several cases of highway driving. We analyze a platoon that consists of several vehicles including SV in a faster target lane and LV in a slower lane that tries to move into the target lane. Note that the focus of this study is on discretionary lane-changing behavior and LV can increase its utility by changing lane into a faster lane. Therefore, it is assumed that LV inevitably cuts into the platoon. For SV to accept LV as a new predecessor, SV should adjust its speed and create a safe distance between the vehicles. After that, SV will try to regain its desired velocity or the platoon speed (SV's speed before the lane-changing maneuver).

The first step of the evaluation process is focused on assessing the effectiveness of the cost functions (Figure 11 and Figure 13) under diverse scenarios by adjusting the cost functions' weights. Accordingly, the weights given here are just test numbers and have been estimated based on try and error. These weights, however, can be optimized for various scenarios and the optimized weights are expected to further improve the performance of MPC. For a general description of how we pick the weight values for simulation in this study, we relate to the initial conditions such as velocity difference of SV and LV and the headway between SV and the old leader in the target lane (TV, red car) in comparison to the desired time headway of SV. Considering how polynomial-based trajectory is generated, those two factors are related to the condition used to generate the lane-changing trajectory and velocity profile. To find the proper value for in a scenario, we first find some values for based on several generic simulations. Afterwards, we interpolate them to obtain estimated proper depending on the initial conditions (i.e., and ). This does not tell accurate values for the weight, but this still can be appropriate estimate depending on .

In such a way and by using different values for the weight , we test the MPC for various scenarios. Accordingly, the MPC can generate a naturalistic car-following reaction to LV. In other scenarios except for what is given in Figure 42, we use 0 for . Other weights used for the simulation are given as:

* The cost function of Figure 11: = 100, = 500
* The cost function of Figure 13: = 70, = 50, = 800, and = 200

A close up of a map

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Figure 42. Diagram. depending on (0-8m/s) and the distance d.

For a comprehensive analysis, we simulate typical scenarios that a lane-changing maneuver is likely to happen. Also, to evaluate the performance of the proposed MPC, we compare the behavior of the platoon when only Swaroop's controller is used (for all of the vehicles in the platoon) with when the proposed MPC is utilized by SV. A comparison is made based on driving comfort and safety. Note that comfort is a critical factor in autonomous driving and therefore, jerk values are utilized in this to represent driver/passenger comfort. Considering the case of all 100% market penetration rate of automated vehicles, we simulate a platoon that only consists of vehicles with Swaroop's controller and compares the results with a platoon with the proposed MPC on SV. Two generic scenarios are tested:

* Maintaining Speed: both lanes have the same speed ( and = 20 m/s)
* Increasing Speed: Current lane speed ( = 15 m/s) and target lane speed ( = 20 m/s

In the simulation results, we check the factors representing the vehicular movements (i.e., velocity, acceleration, and jerk) during the lane-changing maneuver. Figure 43 illustrates the simulation results for the maintaining speed scenario. The red line represents LV, and the blue line represents SV hereafter in all figures. This figure shows the rapid change in acceleration that can cause driving discomfort to a driver regardless of the magnitude of velocity and acceleration when utilizing Swaroop's controller. According to Graaf et al. (Graaf and Van Weperen, 1997), an average human can handle about 0.6 without losing the balance. This value is exceeded in this case. However, In Figure 43, by implementing the MPC, SV smoothly changes its velocity with jerk less than 0.6 , which is noticeable from the figure. Furthermore, we can see that the velocity of SV is affected by both the distance and the acceleration profile of LV. In Figure 44, SV starts to increase its velocity right after LV stops slowing down. In Figure 44, where the lane-changing maneuver happens to obtain speed advantage, we can see that MPC handles the jerking issue very well and shows good performance in terms of adapting to the LV behavior. Moreover, in both cases, we can see that SV starts to recover its velocity with acceleration smaller than LV's acceleration after velocities of LV and SV become equal. Note that the performance is expected to increase by optimizing the weights in the MPC.

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Figure . Diagrams. Maintaining Speed.

Note: Two lane speed = 20 m/s; (left) MPC based platoon, and (right) Swaroop's controller based platoon.

A close up of a map

Description automatically generated

**Source: FHWA**

Figure . Diagrams. Increasing Speed.

Note: Current lane speed = 15 m/s, Target lane speed = 20 m/s; (left) MPC based platoon, and (right) Swaroop's controller based platoon.

Performance

In this section, we evaluate the performance of the optimal lane-changing trajectory by checking if it can decrease the total deceleration effort. Here are the simulation scenarios:

* Lane change duration: = 6s
* Current lane speed ) = 22.5 m/s, Target lane speed () = 25 m/s
* Distance between LV and the target leader: = 0.2 = 5m

We intentionally choose the scenario above where the optimized trajectory is different than the one without the 6th order term. Therefore, we can see later that by adding the 6th order term with a chosen coefficient, the model can make a difference in terms of minimizing the disturbances in the platoon. First, we obtain the range of feasible considering physical constraints. Since we are considering adjusting only the coefficient, which determines behavior in the X-axis, we do not take the lateral acceleration into account as a physical constraint for now. This is reasonable because polynomials for Y-axis behavior does not depend on (according to Papadimitriou et al. (Papadimitriou and Tomizuka, 2003)) but depend on boundary conditions in the Y-axis, (), and the lane change duration . In our research, only is non-zero according to desired Y-axis position after the lane change, and is chosen later to be sufficient considering maximum while a lane change. Given, the range of feasible is obtained. Therefore, we do not take lateral acceleration into account here and we only focus on the longitudinal acceleration of SV as a physical constraint.

Figure 45 shows when the physical constraint is violated for various values of. Therefore, that remains in the blue area along the lane-changing duration axis can be chosen. In the simulation, we set 2.5 and 4.0 as the longitudinal acceleration constraints. Accordingly, two green lines in Figure 45 represent maximum and minimum values that satisfy the constraint, respectively. Moreover, to get the optimal, the model searches the range defined by green lines in Figure 45, which is between -0.005 and 0.0026. By choosing a value inside the blue area, that represents satisfying the physical constraints, we can obtain candidates for the optimal lane-changing trajectory.

The approach tries to minimize the total control effort by predicting the acceleration values in the target lane. Minimizing the total control effort results in less deviation from the desired speed (or equilibrium speed) as shown in Figure 46; thus, minimizing the total control effort results in minimizing total travel time within the platoon. This figure shows the speed and acceleration profiles of SV depending on different values. In the first case with (lower order polynomial), we still can see that MPC properly works so that SV can safely follow LV as shown in the previous simulation. Taking one step further, in the second case, by introducing the sixth order term , the performance of MPC further improves. Even though SV is still affected by the optimal lane-changing trajectory, a smaller shockwave takes place compared with the first case.

A close up of a map

Description automatically generated

**Source: FHWA**

Figure . Diagram. Range of feasible value of α\_6: Constrained by maximum longitudinal acceleration.

A close up of a map

Description automatically generated

**Source: FHWA**

Figure . Diagrams. Lane-changing maneuver profiles (ACC-based Platoon).

Note: α\_6= 0 (lower order polynomial) and α\_6 = 0.0026 (optimal).

The key advantage of the proposed method comes from the use of MPC along with V2V communications. The model can take advantage of using the actual trajectory that LV will follow, not the predicted future behavior of LV based on the measurement (e.g., sensor inputs), which is done by a generic MPC. This can significantly increase the accuracy in terms of how the optimized trajectory will minimize the traffic disruption from the lane change. Accordingly, we also test the scenarios to evaluate the performance of the MPC when additional information is available through V2V communications. In simulations, we made the optimized lane change trajectory profile directly available, and SV used it. Therefore, MPC of SV foresees the future state of LV based on the actual lane-changing maneuver and output desired acceleration.

To see if minimizing total control effort is results in the desired outcome, we did the same simulation with the different platoon members. We considered platoon members based on IDM only, ACC only, and ACC and IDM combined. Figure 47 illustrates the simulation results for various market penetration rates of automated vehicles (i.e., 30%, 50%, and 70%). The results indicate the capability of the presented approach in containing the shockwave due to the lane-changing maneuver. Note that as the market penetration rate of automated vehicle increase, the shockwave magnitude and duration decreases.

A close up of a map

Description automatically generated

**Source: FHWA**

Figure . Diagrams. Left column (Maintaining Speed) and Right column (Increasing Speed).

Note: 30%(first row), 50% (second row) and 70%(third row) of ACC-based cars in the platoon. ACC vehicles are in black and IDM vehicles are in green.

Gap generation

Figure 48 shows how the value of variance constraints can change platoon behavior. Three constrain sets were utilized for this purpose:

* Constraint set 1: = 0.02 and = 0.007.
* Constraint set 2: = 0.0018 and = 0.0065.
* Constraint set 3: = 0.0015 and = 0.0063.







**Source: FHWA**

Figure . Diagrams. Impact of different variance constraints on time headway, velocity, and speed variance of three vehicles ahead of the lane-changing vehicles in the target lane.

Note: h\_desired = 1.0 sec, h\_safety = 0.6 sec, and sum of change of h = 1.0 sec. All the graphs.

Chapter 6. SUMMARY AND RECOMMENDATIONS

This study proposed a novel merge coordination system based on controlling both the lane-changing trajectory and car-following behavior of the vehicles in the target lane. The model was implemented in Northwestern University’s microscopic traffic simulation tools and the simulation results indicated the effectiveness of the proposed methodology in reducing the shockwave magnitude and duration. Multiple sensitivity analysis experiments were also conducted for both fully autonomous and mixed driving environments. The model was successful in mitigating shockwave formation and propagation in almost all cases.

The proposed methodology, however, depends significantly on reliable communications. Vehicle to everything (V2X) communications do not necessarily exist at all times due to signal interference and information loss. An updated model should be developed to consider this lack of information availability. Such a model is critical to ensure a reliable merge coordination system in the real-world.

APPENDIX

The following structure is to predict the impact of a lane-changing maneuver on a platoon when we assume that the communications between vehicles such as V2X communication is available. In detail, the lane-changing vehicle can read information on the controller of its follower in the platoon. With this information, the lane-changing vehicle can predict followers’ behavior in response to a pre-generated lane-changing trajectory using an iterative process.

**Input Arguments**

Lane-changing trajectory boundary conditions: location, velocity, and acceleration of the Lane-changing vehicle in X-axis and Y-axis.

Lane-changing trajectory parameter to test: Sixth order coefficient of X-axis polynomial.

**Output Arguments**

Total deceleration applied to the follower of a lane-changing vehicle: *Impact*

**Pseudo Code**

* If (Lane change begins)
  + Get the trajectory boundary conditions: location, velocity, and acceleration of a lane-changing vehicle in X-axis and Y-axis.
  + Get the trajectory parameter to test: Sixth order coefficient of X-axis polynomial.
  + Generate the lane change trajectory: X-axis polynomial and Y-axis polynomial.
  + Set *Impact* = 0, which represents the total deceleration applied to the potential follower of a lane-changing vehicle during a lane-changing maneuver.
  + While (Keep iterating until the termination condition is met):
    - Compare current velocities of the lane-changing vehicle and its potential follower after a lane-changing maneuver.
    - If (Velocity of the follower > Velocity of lane change car)
      * Predict the behavior of the first Model Predictive controller on the follower, which is designed to slow down for the safety, in response to the pre-generated trajectory: Computes optimal acceleration change.
    - Else:
      * Predict the behavior of the second Model Predictive controller on the follower, which is designed to recover its velocity, in response to the pre-generated trajectory: Computes optimal acceleration change.
    - *Impact* += the optimal acceleration changes of the follower.
    - If (*Impact* ≥ \_0)
      * Return *Impact*
      * Terminates While loop

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The original map in Figure 25 has been modified and is the copyright property of Google® Maps™ and can be accessed from https://www.google.com/maps/.(#) The map overlays show the data collection location on Interstate 35 near Austin, TX.

References

2007. Next Generation Simulation: US101 Freeway Dataset Federal Highway Administration, Washington, D.C.

Arefizadeh, S., Talebpour, A., 2018. A platooning strategy for automated vehicles in the presence of speed limit fluctuations. *Transportation research record* 2672(20), 154-161.

Bacha, A., Bauman, C., Faruque, R., Fleming, M., Terwelp, C., Reinholtz, C., Hong, D., Wicks, A., Alberi, T., Anderson, D., 2008. Odin: Team victortango's entry in the darpa urban challenge. *Journal of field Robotics* 25(8), 467-492.

Cong, Y., Sawodny, O., Chen, H., Zimmermann, J., Lutz, A., 2010. Motion planning for an autonomous vehicle driving on motorways by using flatness properties, *2010 IEEE International Conference on Control Applications*. IEEE, pp. 908-913.

González, D., Pérez, J., Lattarulo, R., Milanés, V., Nashashibi, F., 2014. Continuous curvature planning with obstacle avoidance capabilities in urban scenarios, *17th International IEEE Conference on Intelligent Transportation Systems (ITSC)*. IEEE, pp. 1430-1435.

Graaf, B.D., Van Weperen, W., 1997. The retention of balance: An exploratory study into the limits of acceleration the human body can withstand without losing equilibrium. *Human factors* 39(1), 111-118.

Hamdar, S.H., Treiber, M., Mahmassani, H.S., 2009. Calibration of a stochastic car-following model using trajectory data: exploration and model properties.

Hamdar, S.H., Treiber, M., Mahmassani, H.S., Kesting, A.J.T.r.r., 2008. Modeling driver behavior as sequential risk-taking task. 2088(1), 208-217.

Kianfar, R., Ali, M., Falcone, P., Fredriksson, J., 2014. Combined longitudinal and lateral control design for string stable vehicle platooning within a designated lane, *17th International IEEE Conference on Intelligent Transportation Systems (ITSC)*. IEEE, pp. 1003-1008.

Kim, J., Mahmassani, H.S., 2011. Correlated parameters in driving behavior models: Car-following example and implications for traffic microsimulation. *Transportation research record* 2249(1), 62-77.

Kolski, S., Ferguson, D., Bellino, M., Siegwart, R., 2006. Autonomous driving in structured and unstructured environments, *2006 IEEE Intelligent Vehicles Symposium*. IEEE, pp. 558-563.

Maschuw, J.P., Keßler, G.C., Abel, D., 2008. LMI-based control of vehicle platoons for robust longitudinal guidance. *IFAC Proceedings Volumes* 41(2), 12111-12116.

Milanés, V., Shladover, S.E., 2014. Modeling cooperative and autonomous adaptive cruise control dynamic responses using experimental data. *Transportation Research Part C: Emerging Technologies* 48, 285-300.

Nelson, W., 1989. Continuous-curvature paths for autonomous vehicles, *Proceedings, 1989 International Conference on Robotics and Automation*. IEEE, pp. 1260-1264.

Papadimitriou, I., Tomizuka, M., 2003. Fast lane changing computations using polynomials, *Proceedings of the 2003 American Control Conference, 2003.* IEEE, pp. 48-53.

Piazzi, A., Bianco, C.L., Bertozzi, M., Fascioli, A., Broggi, A., 2002. Quintic G/sup 2/-splines for the iterative steering of vision-based autonomous vehicles. *IEEE Transactions on Intelligent Transportation Systems* 3(1), 27-36.

Pivtoraiko, M., Kelly, A., 2005. Efficient constrained path planning via search in state lattices, *International Symposium on Artificial Intelligence, Robotics, and Automation in Space*. Munich Germany, pp. 1-7.

Pivtoraiko, M., Knepper, R.A., Kelly, A., 2009. Differentially constrained mobile robot motion planning in state lattices. *Journal of Field Robotics* 26(3), 308-333.

Rahmati, Y., Khajeh Hosseini, M., Talebpour, A., Swain, B., Nelson, C., 2019. Influence of Autonomous Vehicles on Car-Following Behavior of Human Drivers. *Transportation Research Record*, 0361198119862628.

Rastelli, J.P., Lattarulo, R., Nashashibi, F., 2014. Dynamic trajectory generation using continuous-curvature algorithms for door to door assistance vehicles, *2014 IEEE Intelligent Vehicles Symposium Proceedings*. IEEE, pp. 510-515.

Reece, D.A., Shafer, S.A.J.T.R.P.A.P., Practice, 1993. A computational model of driving for autonomous vehicles. 27(1), 23-50.

Schroder, J., Gindele, T., Jagszent, D., Dillmann, R., 2008. Path planning for cognitive vehicles using risk maps, *2008 IEEE Intelligent Vehicles Symposium*. IEEE, pp. 1119-1124.

Seiler, P., Pant, A., Hedrick, K., 2004. Disturbance propagation in vehicle strings. *IEEE Transactions on automatic control* 49(10), 1835-1842.

Swaroop, D., 1997. String stability of interconnected systems: An application to platooning in automated highway systems.

Talebpour, A., Mahmassani, H.S., 2016a. Influence of connected and autonomous vehicles on traffic flow stability and throughput. *Transportation Research Part C: Emerging Technologies* 71, 143-163.

Talebpour, A., Mahmassani, H.S., Hamdar, S.H., 2017. Effect of information availability on stability of traffic flow: Percolation theory approach. *Transportation Research Procedia* 23, 81-100.

Talebpour, A., Mahmassani, H.S., Hamdar, S.H.J.T.r.r., 2011. Multiregime sequential risk-taking model of car-following behavior: specification, calibration, and sensitivity analysis. 2260(1), 60-66.

Talebpour, A., Mahmassani, H.S.J.T.R.P.C.E.T., 2016b. Influence of connected and autonomous vehicles on traffic flow stability and throughput. 71, 143-163.

Toledo, T., Zohar, D., 2007. Modeling duration of lane changes. *Transportation Research Record* 1999(1), 71-78.

Treiber, M., Hennecke, A., Helbing, D., 2000. Congested traffic states in empirical observations and microscopic simulations. *Physical review E* 62(2), 1805.

Van Arem, B., Van Driel, C.J., Visser, R.J.I.T.o.i.t.s., 2006. The impact of cooperative adaptive cruise control on traffic-flow characteristics. 7(4), 429-436.

Wang, J., Rajamani, R., 2004. Should adaptive cruise-control systems be designed to maintain a constant time gap between vehicles? *IEEE Transactions on Vehicular Technology* 53(5), 1480-1490.